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OSCILLATORY MHD FREE AND FORCED CONVECTIVE FLOW AND MASS TRANSFR THROUGH A VERTICAL POROUS PLATE IN SLIP-FLOW REGIME WITH VARIABLE SUCTION AND CONSTANT HEAT FLUX

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ABSTRACT

An attempt has been made to study an oscillatory MHD free and forced convective flow past a vertical porous plate in slip-flow regime with variable suction and constant heat flux. The temperature of the plate oscillates in time about a constant mean and a uniform magnetic field is assumed to be applied transversely to the direction of the flow. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The solutions to the governing equations are derived by regular perturbation technique with Eckert number (E) as perturbation parameter.

The expressions for the velocity field, temperature field, skin friction at the plate in the direction of the flow and the plate temperature are obtained in non-dimensional form. The amplitudes and phases of the fluctuating parts of the skin friction and rate of heat transfer (Nusselt Number) are obtained in non-dimensional form. The effects of the Hartmann number (M), the frequency of oscillation(ω), suction parameter(A) and the rarefaction parameter(h) on these fields are discussed and demonstrated with the help of graphs.

KEYWORDS: MHD, Viscous, Oscillatory, Electrically Conducting, Incompressible, Heat Transfer, Slip-Flow, Eckert Number

1. INTRODUCTION

The problems of convective flows arising in fluids as a result of interaction of the force of gravity and density difference caused by simultaneous diffusion of thermal energy and chemical species have been studied by many authors due to applications of such problems in Geophysics and Engineering. In this regard, we may cite the works done by Bejan and Khair [1], Trivision and Bejan [2], Acharya etc al [3], Raptis and Kafousias [4], Ahmed etc al ([5]) . Unsteady oscillatory free and forced convective flows plays a major rule in chemical engineering, turbo machinery and aerospace technology. The temperature change causes density variation which leads to the free convection in the fluid. The free convection flow is enhanced by superimposing oscillating temperature on the mean plate temperature.

Soundalgekar [6,7,8] studied the free convection effects on mean velocity and temperature field of oscillatory flow past an infinite vertical porous plate with constant suction with or without transverse magnetic field. The analysis of viscous dissipation on the transient free convection flow past a semi-infinite vertical plate is studied by Soundalgekar et. al. [9]. The results of the combined free and forced convection flow of water at 4° C from a vertical plate with variable temperature was investigated analytically by Vighnesam and Soundalgekar [10]. Sahoo et. al [11] have investigated an unsteady MHD free convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate subjected to constant suction and heat sink. The analysis of the MHD unsteady free convection flow past a vertical porous plate was made by Anwar $\left[12\right]$. Hussain et. al. $\left[13\right]$ studied the effects of a fluctuating surface temperature and concentration on a natural convection flow from a vertical plate.

Analytical solutions to the problems of the transient free convective viscous incompressible flow past a vertical plate with periodic temperature and variable suction in a slip-flow regime are presented by many authors. Some of them are Sharma and Choudhary $\left[14\right]$, Sharma $\left[15\right]$, Jain and Sharma $\left[16\right]$ and Ahmed and Goswami[17]. This paper presents the extension work of Ahmed and Goswami $\left[17\right]$ to investigate the effects of various parameters namely Hartmann number (M), the frequency of oscillation(ω), suction parameter(A) and the rarefaction parameter(h) on a flow with mass transfer in a slip-flow regime.

2. MATHEMATICAL FORMULATION

We now consider a two dimensional boundary layer flow of an oscillatory MHD free and forced convective flow past a vertical porous plate in slip-flow regime with variable suction in presence of a transverse magnetic field by making the assumptions.

I. All the properties except the density in the buoyancy force term are constant.

II. The Eckert number E is small.

III. The Magnetic Reynolds number is so small that induced magnetic field can be neglected.

IV. The plate is subjected to a normal suction velocity, which varies periodically with time about a constant mean v_0 (say).

V. The Magnetic dissipation term in the energy equation is negligible.

We introduce a coordinate system $\left(\overline{x},\overline{y},\overline{z}\right)$ with X-axis vertically upward along the plate, Y-axis perpendicular to it and directed in to the fluid region and Z-axis perpendicularly to XY plane. Let $\vec{q}=\overline{u}\hat{i}+\overline{v}\hat{j}$ be the fluid velocity at the point ($\left(\overline{x},\overline{y},\overline{z}\right)$ and $\vec{B}=B_0\hat{j}$ be the applied magnetic field with strength B_0, $\hat{i}$ and $\hat{j}$ being the unit vectors along X-axis and Y-axis respectively. Since the plate is of infinite length therefore all the physical quantities except the pressure p are independent of x. Under these assumptions the physical quantities are functions of y and t only.

The equations governing the flow are

Equation of Continuity

$$\frac{\partial\overline{v}}{\partial\overline{y}}=0$$

$$\Rightarrow \overline{v}=-v_0(1+\varepsilon Ae^{i\overline{\omega}\overline{t}}) \tag{2.1}$$

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T}-\bar{T}_\infty) + g\bar{\beta}(\bar{C}-\bar{C}_\infty) + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\sigma}{\rho}B_0^{\,2}(\bar{U}-\bar{u}) \tag{2.2}$$

Energy Equation

$$C_p\left[\frac{\partial \bar{T}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{T}}{\partial \bar{y}}\right] = \frac{k}{\rho}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \nu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{2.3}$$

Species continuity equation

$$\frac{\partial \bar{C}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{C}}{\partial \bar{y}} = D_M\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{2.4}$$

The boundary conditions are

$$\bar{u} = \bar{h}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right),\ \bar{T} = \bar{T}_\omega + \varepsilon(\bar{T}_\omega - \bar{T}_\infty)e^{i\bar{\omega}\bar{t}}\ ,\ \bar{C} = \bar{C}_\omega + \varepsilon(\bar{C}_\omega - \bar{C}_\infty)e^{i\bar{\omega}\bar{t}} \quad \text{at } \bar{y}=0 \tag{2.5}$$

$$\bar{u} \to \bar{U},\ \bar{T} \to \bar{T}_\infty,\ \bar{C} \to \bar{C}_\infty \ \text{at}\ \bar{y} \to \infty \tag{2.6}$$

We introduce the following non-dimensional quantities

$$y = \frac{\bar{y}v_0}{\nu},\ t = \frac{\bar{t}v_0^{\,2}}{\nu},\ \omega = \frac{\nu\bar{\omega}}{v_0^{\,2}},\ u = \frac{\bar{u}}{v_0},\ \theta = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_\omega - \bar{T}_\infty},\ U = \frac{\bar{U}}{v_0},\ \phi = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty},$$

$$G_r = g\beta\nu\frac{\bar{T}_\omega - \bar{T}_\infty}{v_0^{\,3}},\quad G_m = \frac{\nu g\bar{\beta}(\bar{C}_w - \bar{C}_\infty)}{v_0^{\,3}},\ P = \frac{\mu C_p}{k},\quad E = \frac{v_0^{\,2}}{C_p(\bar{T}_\omega - \bar{T}_\infty)},\quad h = \frac{v_0\bar{h}}{\nu},\quad M = \frac{\sigma B_0^{\,2}\nu}{\rho v_0^{\,2}},$$

$$S_c = \frac{\nu}{D_M},\quad S_0 = \frac{1}{\lambda},\ \nu = \frac{\mu}{P}.$$

Where G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, P is the Prandtl number, E is the Eckert number, h is the rarefaction parameter, k is the thermal conductivity, A is the suction parameter, ν is the kinematics viscosity, M is the Hartmann number, C_p is the specific heat at constant temperature, β is the coefficient of thermal expansion, S_c is the Schmidt number, ω is the frequency parameter and the other symbols have their usual meanings.

The non-dimensional form of the equations (2.2), (2.3) and (2.4) are

$$\frac{\partial u}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} = G_r\theta + G_m\phi + \frac{\partial^2 u}{\partial y^2} + M(U-u) \tag{2.7}$$

$$\frac{\partial \theta}{\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{P}\frac{\partial^2 \theta}{\partial y^2} + E\left(\frac{\partial u}{\partial y}\right)^2 \qquad 2.8)$$

$$\frac{\partial\phi}{\partial t}-(1+\varepsilon Ae^{iwt})\frac{\partial\phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2}+S_0\frac{\partial^2\theta}{\partial y^2} \quad 2.9)$$

Subject to the boundary conditions

$$u=h\frac{\partial u}{\partial y},\ \theta=1+\varepsilon e^{i\omega t},\ \phi=1+\varepsilon e^{i\omega t} \text{ at } y=0 \quad 2.10)$$

$$u=U,\ \theta=0,\ \phi=0 \text{ at } y\to\infty \quad 2.11)$$

3. METHOD OF SOLUTION

Assuming the small amplitude of oscillation ($\varepsilon \ll 1$), we represent the velocity u and the temperature θ, near the plate as follows

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+0(\varepsilon^2) \quad (3.1)$$

$$\theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y)+o(\varepsilon^2) \quad (3.2)$$

$$\phi(y,t)=\phi_0(y)+\varepsilon e^{i\omega t}\phi_1(y)+o(\varepsilon^2) \quad (3.3)$$

Substituting the above assumptions in (2.6) to (2.10) and equating the harmonic terms and neglecting ε^2, the equations for u_0, u_1, θ_0 and θ_1 with reduced boundary conditions are as follows.

$$u_0''+u_0'-Mu_0=-G_r\theta_0-G_m\phi_0-MU \quad (3.4)$$

$$u_1''+u_1'-(M+i\omega)u_1=-G_r\theta_1-G_m\phi_1-Au_0' \quad (3.5)$$

$$\theta_0''+P\theta_0'=-EPu_0'^2 \quad (3.6)$$

$$\theta_1''+P\theta_1'-iP\omega\theta_1=-2PEu_0'u_1'-AP\theta_0' \quad (3.7)$$

$$\phi_0''+S_c\phi_0'=-S_0S_c\theta_0'' \quad (3.8)$$

$$\phi_1''+S_c\phi_1'-iP\omega\phi_1=-AS_c\phi_0'-S_0S_c\theta_1'' \quad (3.9)$$

Subject to the boundary conditions:

$$u_0=h\frac{\partial u_0}{\partial y},\ u_1=h\frac{\partial u_1}{\partial y},\ \theta_0=1,\ \theta_1=1,\ \phi_0=1,\ \phi_1=1 \text{ at } y=0 \quad (3.10)$$

$$u_0=U,\ u_1=0,\theta_0=0,\ \theta_1=0,\ \phi_0=0,\ \phi_1=0 \text{ at } y\to\infty \quad (3.11)$$

To solve the equations we assume

$$u_0=u_{01}+Eu_{02}+O(E^2);\ u_1=u_{11}+Eu_{12}+O(E^2) \quad (3.12)$$

$$\theta_0=\theta_{01}+E\theta_{02}+O(E^2);\ \ \theta_1=\theta_{11}+E\theta_{12}+O(E^2) \tag{3.13}$$

$$\phi_0=\phi_{01}+E\phi_{02}+O(E^2);\ \ \phi_1=\phi_{11}+E\phi_{12}+O(E^2) \tag{3.14}$$

Substituting above in equations (3.4) to (3.11) and equating the terms independent of E and the coefficient of E in each equation and neglecting E^2, we obtained the following equations with respective boundary conditions are as follows.

$$u_{01}''+u_{01}'-Mu_{01}=-G_r\theta_{01}-G_m\phi_{01}-MU \tag{3.15}$$

$$u_{02}''+u_{02}'-Mu_{02}=-G_r\theta_{02}-G_m\phi_{02} \tag{3.16}$$

$$u_{11}''+u_{11}'-(M+i\omega)u_{11}=-G_r\theta_{11}-G_m\phi_{11}-Au_{01}' \tag{3.17}$$

$$u_{12}''+u_{12}'-(M+i\omega)u_{12}=-G_r\theta_{12}-G_m\phi_{12}-Au_{02}' \tag{3.18}$$

$$\theta_{01}''+P\theta_{01}'=0 \tag{3.19}$$

$$\theta_{02}''+P\theta_{02}'=-P(u_{01}')^2 \tag{3.20}$$

$$\theta_{11}''+P\theta_{11}'-i\omega P\theta_{11}=-AP\theta_{01}' \tag{3.21}$$

$$\theta_{12}''+P\theta_{12}'-i\omega P\theta_{12}=-2Pu_{01}'u_{11}'-AP\theta_{02}' \tag{3.22}$$

$$\phi_{01}''+P\phi_{01}'=-S_0S_c\theta_{01}'' \tag{3.23}$$

$$\phi_{02}''+S_c\phi_{02}'=-S_0S_c\theta_{02}'' \tag{3.24}$$

$$\phi_{11}''+S_c\phi_{11}'-i\omega S_c\phi_{11}=-AS_c\phi_{01}'-S_0S_c\theta_{11}'' \tag{3.25}$$

$$\phi_{12}''+S_c\phi_{12}'-i\omega S_c\phi_{12}=-AS_c\phi_{02}'-S_0S_c\theta_{12}'' \tag{3.26}$$

Subject to the boundary conditions

$$u_{01}=h\frac{\partial u_{01}}{\partial y},\ u_{11}=h\frac{\partial u_{11}}{\partial y},\ u_{02}=h\frac{\partial u_{02}}{\partial y},\ u_{12}=h\frac{\partial u_{12}}{\partial y}$$

$$\theta_{01}=1,\theta_{02}=0,\ \theta_{11}=1,\theta_{12}=0,\ \phi_{01}=1,\phi_{02}=0,\ \phi_{11}=1,\ \phi_{12}=0 \quad \text{at } y=0$$

And

$$u_{01}=U,\ \ u_{11}=0,\ \ u_{02}=0,u_{12}=0,\theta_{01}=0,\theta_{02}=0,\theta_{11}=0,\theta_{12}=0,\ \ \phi_{01}=0,\phi_{02}=0,\ \ \phi_{11}=0,$$
$$\phi_{12}=0 \text{ at } y\to\infty$$

Solving the equations from (3.15) to (3.26) with subject to the boundary conditions we get

$$\theta_{01} = e^{-Py} \tag{3.27}$$

$$\theta_{11} = B_1 e^{-\lambda_{11} y} + B_2 e^{-Py} \tag{3.28}$$

$$u_{01} = A_{01} e^{-\lambda_1 y} + A_{02} e^{-Py} + A_{03} e^{-S_c y} + U \tag{3.29}$$

$$\theta_{02} = F_{01} e^{-Py} - F_{02} e^{-2\lambda_1 y} - F_{03} e^{-2Py} - F_{04} e^{-2S_c y} - F_{05} e^{-(\lambda_1 + P)y} - F_{06} e^{-(P+S_c)y} - F_{07} e^{-(\lambda_1 + S_c)y} \tag{3.30}$$

$$\phi_{01} = (1 + D_{10}) e^{-S_c y} - D_{10} e^{-Py} \tag{3.31}$$

$$\phi_{11} = D_{11} e^{-Ly} + D_{12} e^{-S_c y} + D_{13} e^{-Py} + D_{14} e^{-\lambda_{11} y} \tag{3.32}$$

$$\phi_{02} = D_{15} e^{-S_c y} + D_{16} e^{-Py} + D_{17} ec^{-2\lambda_1 y} + D_{18} e^{-2Py} + D_{19} e^{-2S_c y} + D_{20} e^{-(P+\lambda_1)y} + D_{21} e^{-(P+S_c)y} + D_{22} e^{-(\lambda_1 + S_c)y} \tag{3.33}$$

$$u_{02} = T_{00} e^{-\lambda_1 y} + T_{01} e^{-Py} + T_{02} e^{-2\lambda_1 y} + T_{03} e^{-2Py} + T_{04} e^{-2S_c y} + T_{05} e^{-(\lambda_1 + P)y} + T_{06} e^{-(P+S_c)y} + T_{07} e^{-(\lambda_1 + S_c)y} + T_{08} e^{-S_c y} \tag{3.34}$$

$$u_{11} = T_{10} e^{-l_1 y} + T_{11} e^{-\lambda_{11} y} + T_{12} e^{-Py} + T_{13} e^{-S_c y} + T_{14} e^{-Ly} + T_{15} e^{-\lambda_1 y} \tag{3.35}$$

$$\begin{aligned}\theta_{12} &= H_{00} e^{-\lambda_{11} y} + H_{11} e^{-(\lambda_1 + l_1)y} + H_{12} e^{-(\lambda_1 + \lambda_{11})y} + H_{13} e^{-(\lambda_1 + P)y} + H_{14} e^{-(\lambda_1 + S)y} + H_{15} e^{-(\lambda_1 + L)y} \\ &+ H_{16} e^{-2\lambda_1 y} + H_{17} e^{-(P+l_1)y} + H_{18} e^{-(P+\lambda_{11})y} + H_{19} e^{-2Py} + H_{20} e^{-(S_c + P)y} + H_{21} e^{-(P+L)y} + \\ &H_{22} e^{-(S_c + l_1)y} + H_{23} e^{-(S_c + \lambda_{11})y} + H_{24} e^{-2S_c y} + H_{25} e^{-(S_c + L)y} + H_{26} e^{-Py}\end{aligned} \tag{3.36}$$

$$\begin{aligned}\phi_{12} &= D_{23} e^{-Ly} + D_{24} e^{-S_c y} + D_{25} e^{-Py} + D_{26} e^{-2\lambda_1 y} + D_{27} e^{-2Py} + D_{28} e^{-2S_c y} \\ &+ D_{29} e^{-(\lambda_1 + P)y} + D_{30} e^{-(S_c + P)y} + D_{31} e^{-(\lambda_1 + S_c)y} + D_{32} e^{-\lambda_{11} y} + D_{33} e^{-(\lambda_1 + l_1)y} + D_{34} e^{-(\lambda_1 + \lambda_{11})y} + \\ &D_{35} e^{-(\lambda_1 + L)y} + D_{36} e^{-(l_1 + P)y} + D_{37} e^{-(\lambda_{11} + P)y} + D_{38} e^{-(P+L)y} + D_{39} e^{-(S_c + l_1)y} + D_{40} e^{-(S_c + \lambda_{11})y} + D_{41} e^{-(S_c + L)y}\end{aligned} \tag{3.37}$$

$$\begin{aligned}u_{12} &= V_{00} e^{-l_1 y} + V_{11} e^{-\lambda_{11} y} + V_{12} e^{-(\lambda_1 + l_1)y} + V_{13} e^{-(\lambda_1 + \lambda_{11})y} + V_{14} e^{-(\lambda_1 + P)y} + V_{15} e^{-(\lambda_1 + S_c)y} + \\ &V_{16} e^{-(\lambda_1 + L)y} + V_{17} e^{-2\lambda_1 y} + V_{18} e^{-(P+l_1)y} + V_{19} e^{-(P+\lambda_{11})y} + V_{20} e^{-(2P)y} + V_{21} e^{-(P+S_c)y} + V_{22} e^{-(P+L)y} \\ &+ V_{23} e^{-(S_c + l_1)y} + V_{24} e^{-(S_c + \lambda_{11})y} + V_{25} e^{-2S_c y} + V_{26} e^{-(S_c + L)y} + V_{27} e^{-Py} + V_{28} e^{-Ly} + V_{29} e^{-S_c y} + V_{30} e^{-\lambda_1 y}\end{aligned} \tag{3.38}$$

Substituting equations (3.27) to (3.38) in (3.1) and (3.2) we get the expressions for velocity and temperature profiles as given below.

$$u(y,t) = u_{01}(y) + E u_{02}(y) + \varepsilon (u_{11} + E u_{12})(\cos \omega t + i \sin \omega t) \tag{3.39}$$

$$\theta(y,t) = \theta_{01}(y) + E\theta_{02}(y) + \varepsilon (\theta_{11} + E\theta_{12})(\cos \omega t + i \sin \omega t) \tag{3.40}$$

Where $\lambda_1 = \dfrac{1+\sqrt{1+4M}}{2}$, $n_1 = \dfrac{P+\sqrt{P^2+i\omega P}}{2}$, $l_1 = \dfrac{1+\sqrt{1+4M+i\omega}}{2}$,

$$B_1 = 1 - B_2,\ B_2 = i\frac{AP}{\omega},\ B_3 = \frac{G_r(1+Ph) - U(P^2-P-M)}{(1+h\lambda_1)(P^2-P-M)},\ B_4 = \frac{-G_r}{P^2-P-M}$$

The other constant are obtained but not presented here for sake of brevity.

4. COEFFICIENT OF SKIN-FRICTION

The non-dimensional form of Skin-friction at the plate $y = 0$ is given by

$$\tau_0 = \left[\frac{\mu\frac{\partial\bar{u}}{\partial\bar{y}}}{\rho v_0^2}\right]_{\bar{y}=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon e^{i\omega t}u_1'(0) = \tau_0^0 + \varepsilon e^{i\omega t}\tau_0^1$$

$$= \tau_0^0 + \varepsilon\,|B|\cos(\omega t + \alpha)$$

Where $|B| = \sqrt{U_R^2 + U_I^2}$, $\tan\alpha = \dfrac{U_I}{U_R}$

U_R = Real parts of $\left(u_{11}'(0) + Eu_{12}'(0)\right)$, U_I = Imaginary parts of $\left(u_{11}'(0) + Eu_{12}'(0)\right)$

The expressions for U_R and U_I are obtained, but not presented here for the sake of brevity.

5. COEFFICIENT OF HEAT-TRANSFER

The non-dimensional form of rate of heat transfer (Nusselt number N_u) at the plate is given by

$$N_u = -\frac{k}{\rho v_0 C_p(\bar{T}_\omega - \bar{T}_\infty)}\left(\frac{\partial\bar{T}}{\partial\bar{y}}\right)_{\bar{y}=0} = -\frac{1}{P}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\frac{1}{P}\left[\frac{\partial\theta_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial\theta_1}{\partial y}\right]_{y=0}$$

$$= -\frac{1}{P}\theta_0' + \varepsilon\,|H|\cos(\omega t + \beta)$$

Where $|H|^2 = H_R^2 + H_I^2$ and $\tan\beta = \dfrac{H_I}{H_R}$

H_R and H_I are respective real and imaginary parts of $-\dfrac{1}{P}\left(\dfrac{\partial\theta_1}{\partial y}\right)_{y=0}$

6. RESULTS AND DISCUSSIONS

The graphs for amplitude|B| (versus ω and h), the phase tanα (versus ω and h) of fluctuating part of the non-dimensional skin friction τ and the amplitude |H| (versus ω and h), the phase tanβ(versus h) of the fluctuating part of the non-dimensional rate of heat transfer N_u have been presented. Throughout our investigation the Prandtl number P is taken to be equal to 0.71 which corresponds to the air, S=0.22 for H_2, the free steam velocity U is equal to 1, the Grashof number for heat transfer G_r =5, the Grashof number for mass transfer G_m =2 and the Eckert number E is assumed to be 0.05 .The values of other parameters chosen arbitrary.

Figures1and 2 respectively demonstrate the behaviour of $|B|$ against frequency (ω) for different values of Hartmann number (M) and suction parameter (A). Both figures indicate that $|B|$ sharply decreases as ω increases, for small values of ω and then it decreases steadily and slowly. Moreover an increase in the values of (M) causes $|B|$ to decrease and the graphs are distinguished for small and moderate values of M, whereas opposite characteristics are noticed in the second figure.

It is seen from figures 3 and 4 that the effects of Hartmann number (M), suction parameter (A) on $|H|$respectively are almost similar to the effects of these two parameter on $|B|$ in fig1 and fig2.

Figure 5 exhibits the variation of tanα against h . It is observed that for $\omega \leq 5$ the phase (tanα) increases sharply and for large values of ω it is insignificant It is also inferred from this figure that tanα $\to 0$ as $\omega \to \infty$ irrespective of the values of M .

Figures 6 and 7 are demonstrated the variation of tan α against h . It is observed from figure 6 that an increase in the values of h causes $\tan\alpha$ increases for small values of M and it remains constant for moderate values of M . It is also marked from figure 7 that $\tan\alpha$ decreases and the graphs are away from each other as h and M both increases. Further from figure 6 it is inferred that $\tan\alpha$ is distinguished initially for different values of M and as $h \to \infty$ it is insignificant and these behaviour takes reverse trend in figure 7.

Figure8 exhibits the variation of tanβ under the influence of rarefaction parameter h and the Hartmann number M . It is clear from the figure that an increase in causes h tanβ increases as M increases but for small values of M causes tanβ remains constant. It is also noticed from the figure that there is no influence of rarefaction parameter h on tanβ for small values o M . The same figure also shows that for a fixed value of h an increase in the values of M causes a growth in tanβ.

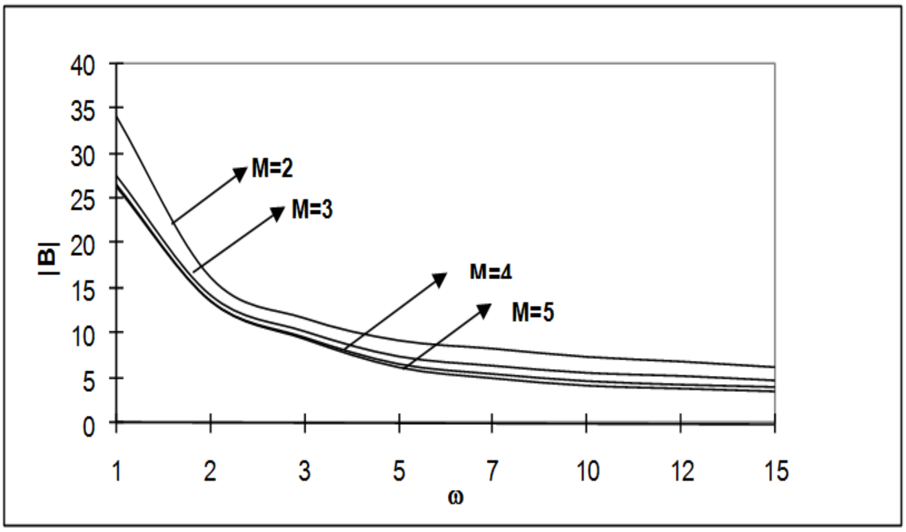

Figure 1: Amplitude (|B|) of Skin Friction Versus Frequency (Ω) When A =2 and h =0.4

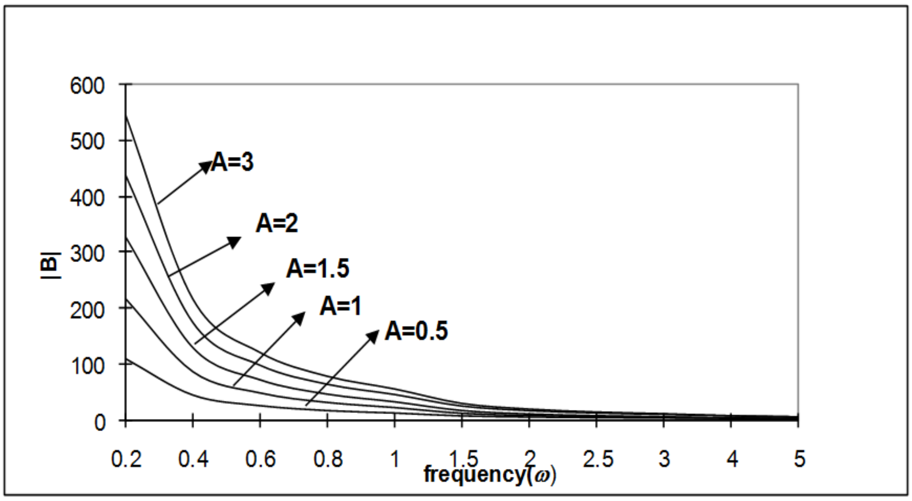

Fig2: Amplitude of Skin-Friction |B| Against Frequency Parameter Ω When M =1

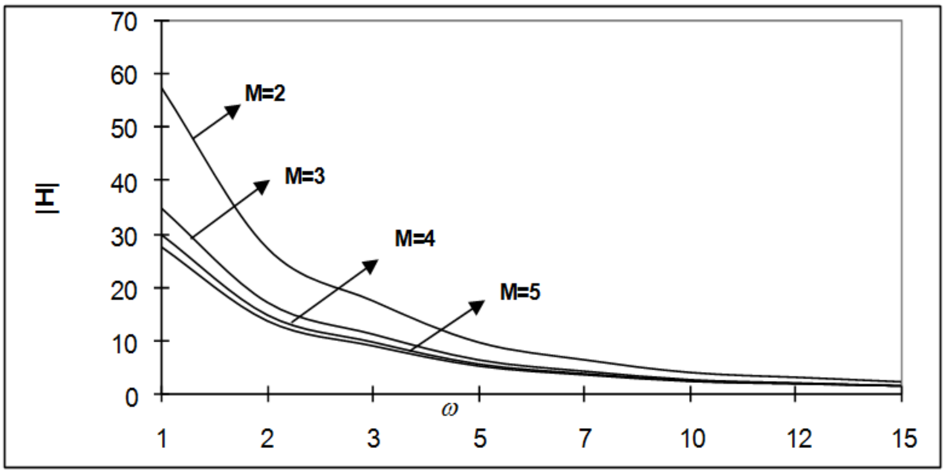

Figure 3: Amplitude of Heat Transfer (|H|) Against Frequency (Ω) When A =2 and h =0.4

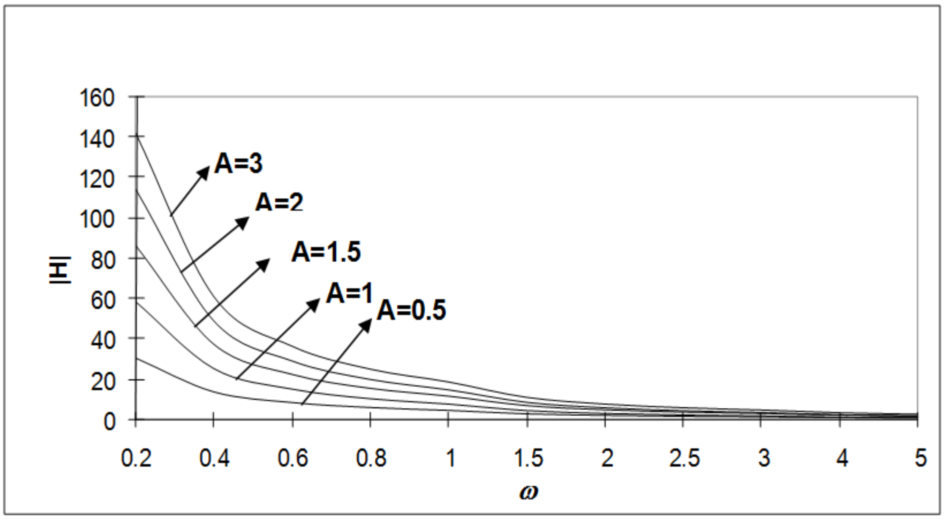

Figure 4: Fluctuation Part of Plate Temperature |H| Against Frequency Parameter Ω When M =1 and h =0.4

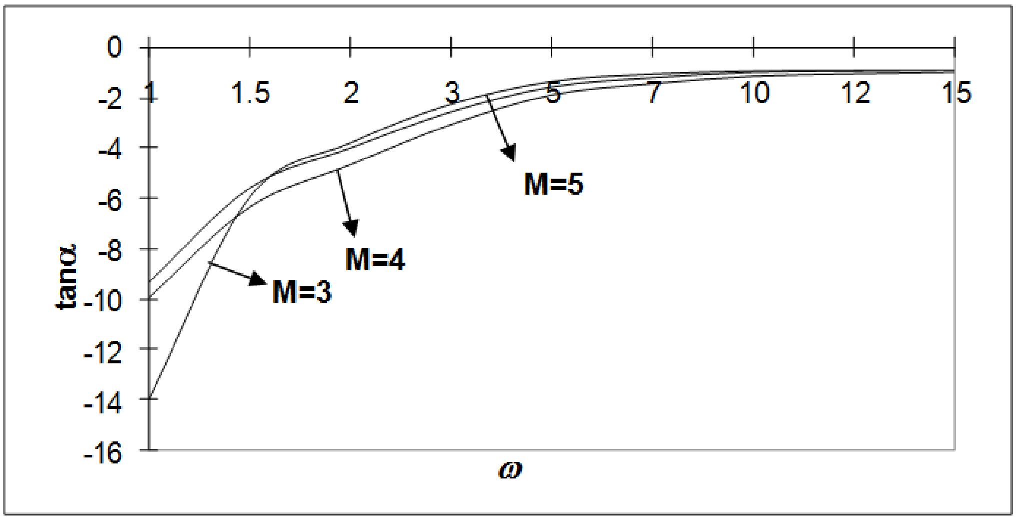

Figure 5: Phase of Skin Friction (Tanα) Against Frequency (Ω) When A =1 and h =0.4

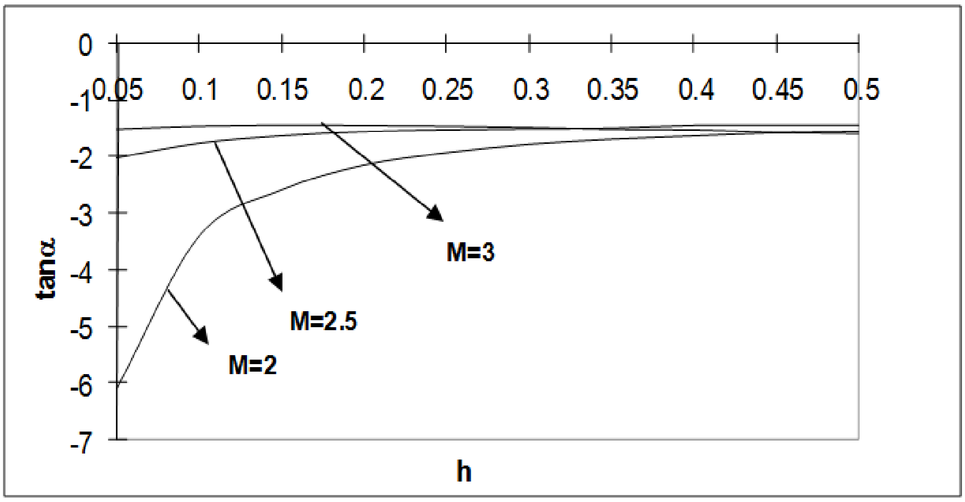

Figure 6: Phase of Skin Friction (Tanα) Versus Rarefaction Parameter (h) When A =2 and ω =3

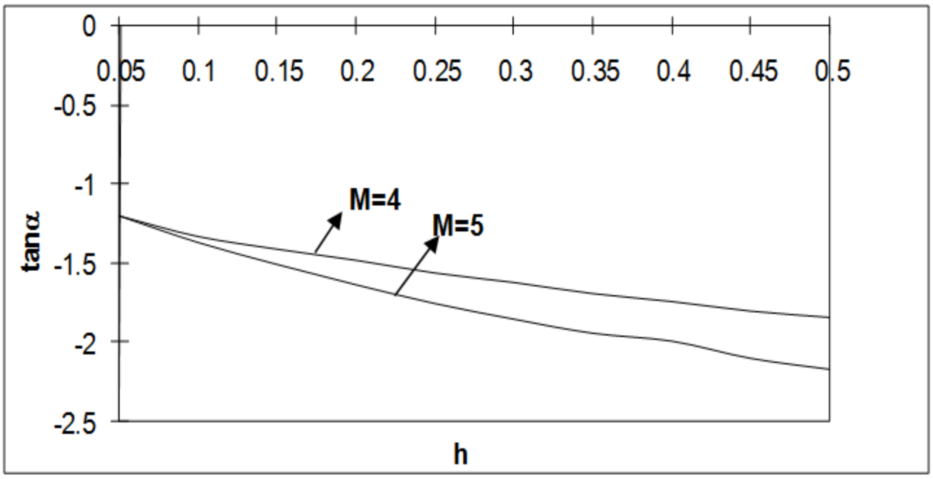

Figure 7: Phase of Skin Friction (Tanα) Against Rarefaction Parameter (h) When A =1 and ω =3

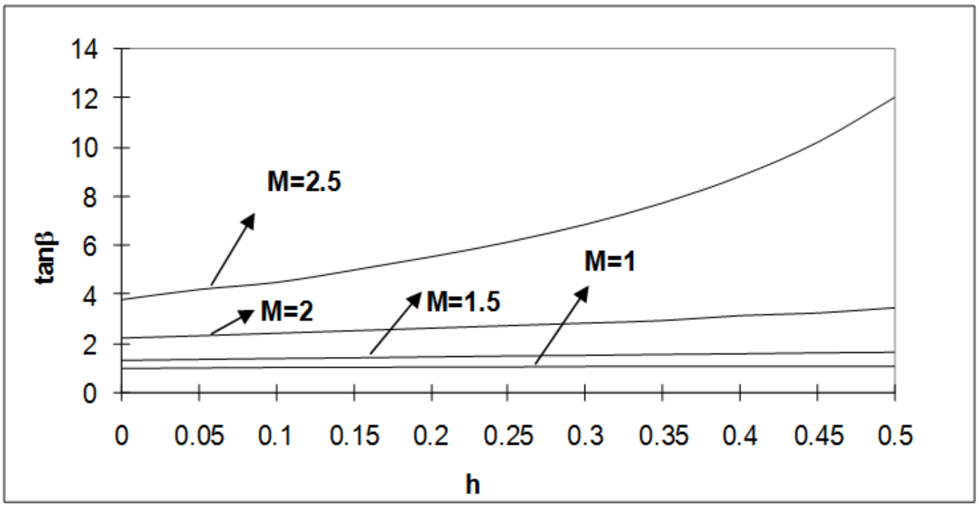

Figure 8: Heat Transfer Phase Tanβ versus Rarefaction Parameter (h) When A =1 and ω =3

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